Unit 6: Polynomial Functions Study Guide Name: Key

1. Find all roots of the polynomial $f(x) = x^3 + x^2 - 2 = 0$ and determine the multiplicity of each. Justify your zeros using factoring and/or synthetic division.

p: -2, ±1, ±2
q: 1, ±1
p/q: ±1, ±2

1	1	1	0	-2
	↓	1	2	2
	1	2	2	0

$x^2 + 2x + 2 = 0$

$\frac{-(2) \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)}$

$\frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2}$

Roots	Multiplicity
1	1
$-1+i$	1
$-1-i$	1

2. Find all real zeros of the polynomial $f(x) = x^4 + 8x^3 + 7x^2$ and determine the multiplicity of each. Justify your zeros using factoring and/or synthetic division.

$x^2(x^2 + 8x + 7) = 0$

$x^2(x+7)(x+1) = 0$

$x^2 = 0 \quad x+7 = 0 \quad x+1 = 0$

Roots	Multiplicity
0	2
-7	1
-1	1

3. Using a graphing utility, graph $f(x) = x^5 - 16x^4 + 64x^3$ and approximate the zeros and their multiplicity. Justify your zeros using factoring and/or synthetic division.

$x^3(x^2 - 16x + 64) = 0$

$x^3(x-8)(x-8) = 0$

$x^3 = 0 \quad x-8 = 0 \quad x-8 = 0$

Roots	Multiplicity
0	3
8	2

4. Write the simplest polynomial (in factored form) with the given zeros and end behavior

Zeros: 4 multiplicity 2, -2 multiplicity 1 $y = -(x-4)^2(x+2)$

End Behavior: $x \to \infty, f(x) \to -\infty$ ↓
$x \to -\infty, f(x) \to \infty$ ↑ ← Represents a negative Leading Coefficient

5. Write the simplest polynomial (in standard form) with the given zeros and end behavior $y = -(x+2)(x-5)(x-\sqrt{3})(x+\sqrt{3})$ → same

Zeros: -2, 5, $\sqrt{3}$, $-\sqrt{3}$

$y = (x^2 - 3x - 10)(x^2 - 3) \to -(x^4 - 3x^2 - 3x^3 + 9x - 10x^2 + 30)$

$-(x^4 - 3x^3 - 13x^2 + 9x + 30)$

End Behavior: $x \to \infty, f(x) \to -\infty$ ↓
$x \to -\infty, f(x) \to -\infty$ ↓

$y = -x^4 + 3x^3 + 13x^2 - 9x - 30$

6. Describe the end behavior of each polynomial function:

A) $f(x) = -2x^4 + 2x^3 + 3x^2 + 4$ ↓ ↓

Negative Leading Coefficient / Even degree

$x \to \infty, f(x) \to -\infty$
$x \to -\infty, f(x) \to -\infty$

B) $f(x) = -2x^5 + 2x^3 + 5$ ↑ ↓

Negative Leading Coefficient / odd degree

$x \to \infty, f(x) \to -\infty$
$x \to -\infty, f(x) \to \infty$

$x = 2/7$ (mult. 2)

$x = 2$

7. If $x = \frac{2}{7}$ is a root of $49x^3 - 126x^2 + 60x - 8 = 0$, use synthetic division to factor the polynomial completely and list all real solutions of the equation.

$(x - \frac{14}{7})(x - \frac{2}{7})$

$(x-2)(7x-2) = 0$

$\frac{2}{7}$	49	−126	60	−8
	↓	14	−32	8
	49	−112	28	0

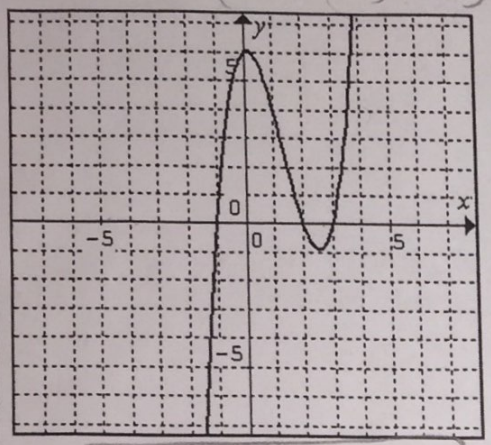
$49x^2 - 112x + 28 = 0$

$7(7x^2 - 16x + 4) = 0$

8. Write a polynomial function ***(in factored and standard form)*** given the following graphs.

A) $y = (x+1)(x-2)(x-3)$

$(x^2 - x - 2)(x-3) = x^3 - 3x^2 - x + 3x - 2x + 6$

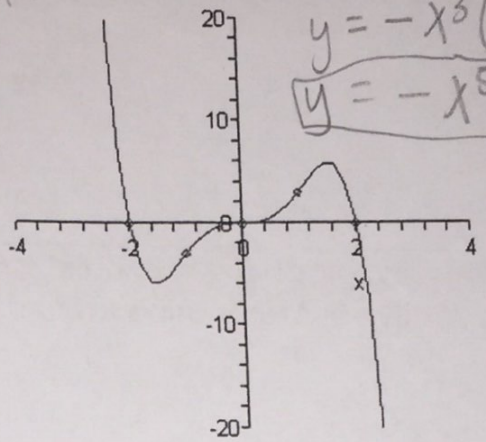


$y = x^3 - 4x^2 + x + 6$

B) $y = -x^3(x+2)(x-2)$

$y = -x^3(x^2 - 4)$

$y = -x^5 + 4x^3$



9. List all possible rational roots using the Rational Root Theorem of the following polynomial.

$P(x) = -10x^3 + x^2 - 22x - 40$

P: −40 ; $\pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$

q: −10 ; $\pm 1, \pm 2, \pm 5, \pm 10$

$\pm \frac{5}{2}, \pm \frac{8}{5}$

P/q: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40, \pm \frac{1}{2}, \pm \frac{1}{5}, \pm \frac{1}{10}, \pm \frac{2}{5}, \pm \frac{4}{5}$

10. $f(x) = 2x^3 - 4x^2 - 2x + 4$

relative maximum(s): $(-.22, 4.2)$

relative minimum(s): $(1.5, -1.3)$

11. $f(x) = -4x^3 - 7x^2 + 8$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Even or Odd Function? Neither

12. $f(x) = 3x^4 - 3x^3 - 3x^2 + 3x$

Intervals of Increase: $(-.6, .4)\ (1, \infty)$

Intervals of Decrease: $(-\infty, -.6)\ (.4, 1)$

13. In July 2005, professional skateboarder jumped over the Great Wall of China. His path can be modeled by the relation $h(d) = -0.05d^2 + 1.15d$, where h is the height above the Great Wall and d is the horizontal distance from the take – off ramp, both in meters. What was Danny's maximum height above the Great Wall?

$(11.5, 6.6)$