

## 8.6 Solving for a Missing Dimension

Given the volume of a shape, we can solve for a missing dimension such as the height or radius. It should come as no surprise that to isolate the variable in the equation, we will use inverse operations.

### Solving for the Height

Let's start with a cylinder with a volume of approximately  $314 \text{ in}^3$  and a radius of  $5 \text{ in}$ . Since we know the formula for the volume of the cylinder, we can plug in and work backwards.

$$\begin{aligned} V &= \pi r^2 h \\ 314 &= 3.14 * (5)^2 h && \text{Substitute what we know} \\ 314 &= 3.14 * 25 * h \\ 314 &= 78.5h && \text{Simplify} \\ \frac{314}{78.5} &= \frac{78.5h}{78.5} \\ 4 &= h && \text{Solve} \end{aligned}$$

So the height of the cylinder is  $4 \text{ in}$ . Similarly, we can solve for the height if we know the volume of a cone. The difference is only the fraction. In most cases it will be easier to eliminate the fraction at the beginning. For example, consider a cone with a volume of  $37.68 \text{ in}^3$  and a radius of  $2 \text{ in}$ . Follow the same process as above to solve for the height.

$$\begin{aligned} V &= \frac{1}{3} \pi r^2 h \\ 37.68 &= \frac{1}{3} * 3.14 * (2)^2 h && \text{Substitute what we know} \\ 3 * 37.68 &= 3 * \frac{1}{3} * 3.14 * (2)^2 h \\ 113.04 &= 3.14 * 4 * h && \text{Multiply by 3 to} \\ &&& \text{eliminate the fraction.} \\ 113.04 &= 12.56h \\ \frac{113.04}{12.56} &= \frac{12.56h}{12.56} \\ 9 &= h && \text{Solve} \end{aligned}$$

## Solving for the Radius

When solving for the radius, we'll have to think back to our knowledge of square and cube roots. Since the radius is either squared or cubed in the volume formulas, we will need to apply either the square root or cube root as one of our inverse operations.

Let's look at a sphere with a volume of  $904.32 \text{ m}^3$ . We know the formula for volume, so we will substitute, simplify and solve.

$$V = \frac{4}{3}\pi r^3$$
$$904.32 = \frac{4}{3} * 3.14 * r^3$$

Substitute what we know

$$\frac{3}{4} * 904.32 = \frac{3}{4} * \frac{4}{3} * 3.14 * r^3$$
$$678.24 = 3.14 * r^3$$

Multiply by  $\frac{3}{4}$  to eliminate the fraction.

$$\frac{678.24}{3.14} = \frac{3.14 * r^3}{3.14}$$
$$216 = r^3$$

Inverse Operations

$$\sqrt[3]{216} = \sqrt[3]{r^3}$$
$$6 = r$$

In this case the radius was  $6 \text{ m}$ , and one of the steps necessary in solving this was using the cube root. If we were solving for the radius in a cylinder or cone, we would need the square root.

## Lesson 8.6

**Answer the following questions using  $\pi \approx 3.14$ . Round your answer to the nearest hundredth where necessary.**

1. Find the height of a cylinder with a volume of  $30 \text{ in}^3$  and a radius of  $1 \text{ in}$ .
2. Find the height of a cylinder with a volume of  $100 \text{ cm}^3$  and a radius of  $2 \text{ cm}$ .
3. Find the height of a cylinder with a volume of  $720\pi \text{ ft}^3$  and a radius of  $6 \text{ ft}$ .
4. Find the height of a cylinder with a volume of  $1215\pi \text{ mm}^3$  and a radius of  $9 \text{ mm}$ .
5. Find the radius of a cylinder with a volume of  $950 \text{ in}^3$  and a height of  $10 \text{ in}$ .
6. Find the radius of a cylinder with a volume of  $208 \text{ cm}^3$  and a height of  $4 \text{ cm}$ .
7. Find the radius of a cylinder with a volume of  $108\pi \text{ ft}^3$  and a height of  $12 \text{ ft}$ .
8. Find the radius of a cylinder with a volume of  $686 \text{ mm}^3$  and a height of  $14 \text{ mm}$ .

9. Find the height of a cone with a volume of  $150 \text{ in}^3$  and a radius of  $10 \text{ in}$ .
10. Find the height of a cone with a volume of  $21 \text{ ft}^3$  and a radius of  $4 \text{ ft}$ .
11. Find the radius of a cone with a volume of  $175 \text{ cm}^3$  and a height of  $21 \text{ cm}$ .
12. Find the radius of a cone with a volume of  $196\pi \text{ mm}^3$  and a height of  $12 \text{ mm}$ .
13. Find the radius of a sphere with volume  $\approx 113.04 \text{ in}^3$ .
14. Find the radius of a sphere with volume  $\approx 904.32 \text{ cm}^3$ .
15. Find the radius of a sphere with volume  $\approx 3052.08 \text{ m}^3$ .
16. Find the radius of a sphere with volume  $\approx 4.18\bar{6} \text{ ft}^3$ .