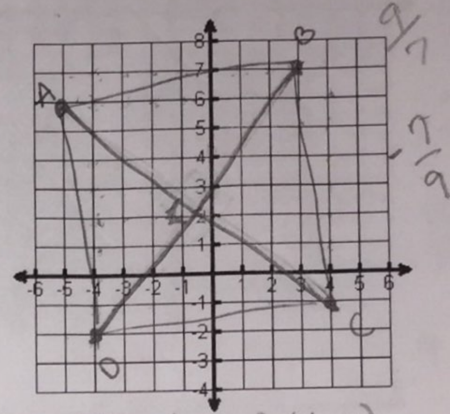
Coordinate Algebra | Unit 6 – Connecting Algebra Through Geometry in the Coordinate Plane | Proving Quads - Day 63

Name: Key Date: ______________

Connecting Algebra & Geometry Through Coordinates

The goal of this assignment is to use the distance and slope formulas to prove statements about geometric figures on the coordinate plane. Since the purpose is to prove a statement, you **must show work.**

1. **Quadrilateral 1**: Plot and label each point. **A(-5, 6), B(3, 7), C(4, -1),** and **D(-4, -2).**



2. *Definition: A parallelogram is a quadrilateral with two pairs of opposite sides that are parallel.* Using the definition of parallelogram, prove that Quadrilateral 1 is a parallelogram.

Slope of AD = -8 Slope of AB = 1/8

Slope of BC = -8 Slope of DC = 1/8

3. *Theorem: A parallelogram with four right angles is a rectangle.* Using the theorem, prove that Quadrilateral 1 is a rectangle.

Slope of AD = -8 ; Slope of AB = 1/8 $\perp$

Slope of BC = -8 ; Slope of DC = 1/8 $\perp$

4. *Definition: A rhombus is a parallelogram with all sides congruent.* Using the definition, prove that Quadrilateral 1 is a rhombus.

$\overline{AB} = (-5,6)(3,7) = \sqrt{(3+5)^2+(7-6)^2} = \sqrt{64+1} = \sqrt{65}$

$\overline{AD} = (-5,6)(-4,-2) = \sqrt{(-4+5)^2+(-2-6)^2} = \sqrt{1+64} = \sqrt{65}$

$\overline{BC} = (3,7)(4,-1) = \sqrt{(4-3)^2+(-1-7)^2} = \sqrt{1+64} = \sqrt{65}$

$\overline{DC} = (-4,-2)(4,-1)$

$\sqrt{(4+4)^2+(-1+2)^2}$

$\sqrt{64+1} = \sqrt{65}$

5. *Definition: A square is a rectangle and a rhombus.* Using the definition, is Quadrilateral 1 a square? Why?

yes, bc all angles are 90° and all sides are congruent

6. *Theorem: The diagonals in a rhombus are perpendicular.* Prove that the theorem is true for Quadrilateral 1.

slope of $\overline{AC}$ = -7/9 slope of BD = 9/7

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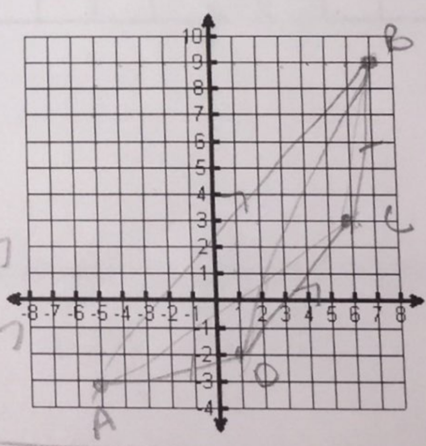
7. **Quadrilateral 2**: Plot and label each point. **A(-5, -3), B(7, 9), C(6, 3),** and **D(1, -2).**



8. *Definition: A trapezoid is a quadrilateral with one pair of opposite sides that are parallel.* Using the definition of trapezoid, prove that Quadrilateral 2 is a trapezoid.

DC = 1 slopes are both 1

AB = 1

9. *Definition: An isosceles trapezoid is a quadrilateral with one pair of opposite sides congruent.* Using the definition of trapezoid, prove that Quadrilateral 2 is an isosceles trapezoid.

$\overline{BC} = (7,9)(6,3)\ \sqrt{(6-7)^2+(3-9)^2} = \sqrt{1+36} = \sqrt{37}$

$AD = (-5,-3)(1,-2)\ \sqrt{(1+5)^2+(-2+3)^2} = \sqrt{36+1} = \sqrt{37}$

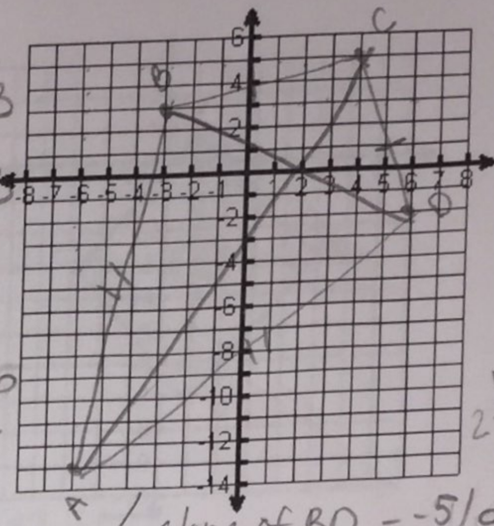
10. *Theorem: The diagonals in an isosceles trapezoid are congruent.* Prove that the theorem is true for Quadrilateral 2.

$\overline{BD} = (7,9)(1,-2) = \sqrt{(1-7)^2+(-2-9)^2} = \sqrt{36+121}\ \ \sqrt{157}$

$\overline{AC} = (-5,-3)(6,3) = \sqrt{(6+5)^2+(3+3)^2} = \sqrt{121+36} = \sqrt{157}$

11. **Quadrilateral 3:** Plot and label each point. **A(-6, -13), B(-3, 3), C(4, 5)**, and **D(6, -2).**

12. *Definition: A kite is a quadrilateral with two pair of consecutive sides that are congruent.* Using the definition of a kite, prove that Quadrilateral 3 is a kite.



$\overline{BC} \cong \overline{CD} =$

$\overline{BC} = (-3,3)(4,5) = \sqrt{(4+3)^2+(5-3)^2} = \sqrt{49+4} = \sqrt{53}$

$\overline{CD} = (4,5)(6,-2) = \sqrt{(6-4)^2+(-2-5)^2} = \sqrt{4+49} = \sqrt{53}$

13. *Theorem: The diagonals of a kite are perpendicular.* Prove that the theorem is true for Quadrilateral 3.

$\overline{BA} \cong \overline{DA}$ $\quad BA\ (-3,3)(-6,-13) = \sqrt{(-6+3)^2+(-13-3)^2} = \sqrt{9+256}$ $\sqrt{265}$

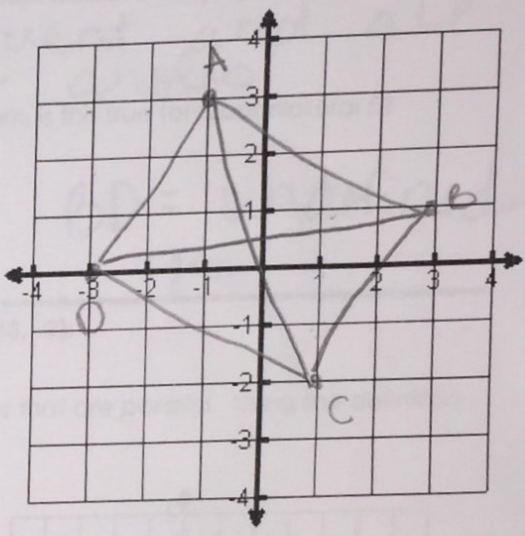
$\overline{DA} = (6,-2)(-6,-13) = \sqrt{(-6-6)^2+(-13+2)^2} = \sqrt{144+121}$ $\sqrt{265}$

slope of BD $= -5/9$

slope of AC $= 9/5$

14. **Quadrilateral 4:** Plot and label each point. **A(-1, 3), B(3, 1), C(1, -2)**, and **D(-3, 0).**

15. *Definition: A parallelogram is a quadrilateral with two pair of opposite sides that are parallel.* Using the definition of a parallelogram, prove that Quadrilateral 4 is a parallelogram.



AD ∥ BC

AD $= 3/2$

BC $= 3/2$

AB ∥ DC

AB $= -1/2$

DC $= -1/2$

16. *Definition: A rectangle is a parallelogram with four right angles.* Using the definition of a rectangle, prove that Quadrilateral 4 is **NOT** a rectangle.

AB ⊥ AD

AB $= -1/2$ Not opp. reciprocals

AD $= 3/2$

BC ⊥ DC

BC $= 3/2$

DC $= -1/2$

17. *Definition: A rectangle is a parallelogram with congruent diagonals.* Using the definition of a rectangle, prove that Quadrilateral 4 is **NOT** a rectangle.

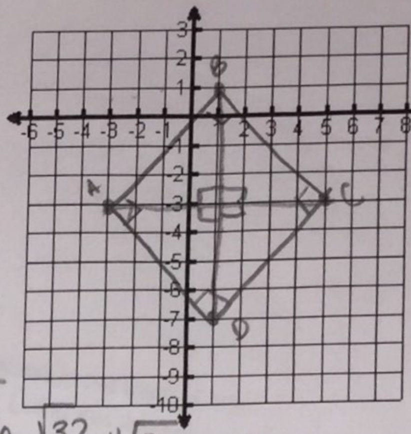
$\sqrt{29} \neq \sqrt{37}$

$\overline{AC} \cong \overline{BD}$?

$\overline{AC}\ (-1,3)(1,-2) = \sqrt{(1+1)^2+(-2-3)^2} = \sqrt{4+25} = \sqrt{29}$

$\overline{BD}\ (3,1)(-3,0) = \sqrt{(-3-3)^2+(0-1)^2} = \sqrt{36+1} = \sqrt{37}$

18. Quadrilateral 5: Plot and label each point. **A(-3, -3)**, **B(1, 1)**, **C(5, -3)**, and **D(1, -7)**.



19. *Definition:* *A parallelogram is a quadrilateral with two pairs of opposite sides that are parallel.* Using the definition of a parallelogram, prove that Quadrilateral 5 is a parallelogram.

AB ∥ DC   AD ∥ BC

AB = 1   DC = 1 ✓   AD = −1   BC = −1 ✓

20. *Definition:* *A rectangle is a parallelogram with 4 right angles.* Using the definition, prove that Quadrilateral 5 is a rectangle.

AB ⊥ AD   BC ⊥ DC

AB = 1   AD = −1 ✓   BC = −1   DC = 1 ✓

21. *Definition:* *A rhombus is a parallelogram with all sides congruent.* Using the definition, prove that Quadrilateral 5 is a rhombus.

$\overline{DC} = \overline{AB} = (-3,-3)(1,1) \quad \sqrt{(1+3)^2+(1+3)^2} = \sqrt{16+16} \quad \sqrt{32} \quad 4\sqrt{2}$ ✓

16 2

$\overline{BC} = \overline{AD} = (-3,-3)(1,-7) \quad \sqrt{(1+3)^2+(-7+3)^2} = \sqrt{16+16} = 4\sqrt{2}$ ✓

22. *Definition:* *A square is a rectangle and rhombus.* Using the definition, is Quadrilateral 5 a square? Why?

yes, all sides are congruent and all angles are right angles

23. *Theorem:* *The diagonals in a rhombus are perpendicular.* Using the theorem, is this true for Quadrilateral 5?

AC ⊥ BD

AC = 0 ✓   BD = undefined ✓

yes

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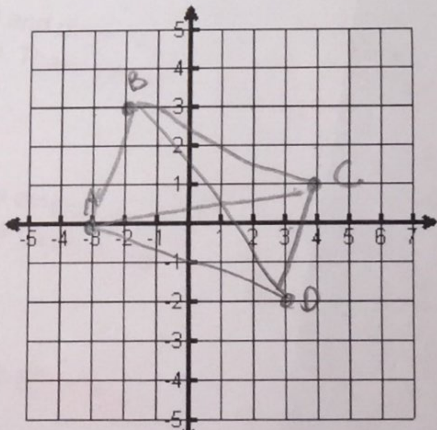
24. **Quadrilateral 6:** Plot and label each point. **A(-3, 0)**, **B(-2, 3)**, **C(4, 1)**, and **D(3, -2)**.



25. *Definition:* *A parallelogram is a quadrilateral with two pairs of opposite sides that are parallel.* Using the definition of a parallelogram, prove that Quadrilateral 6 is a parallelogram.

AB ∥ DC   BC ∥ AD

AB = 3   DC = 3   BC = −1/3   AD = −1/3

26. *Definition:* *A parallelogram with 4 right angles is a rectangle.* Using the definition, prove that Quadrilateral 6 is a rectangle.

AB ⊥ AD   BC ⊥ CD

AB = 3   AD = −1/3   BC = −1/3   CD = 3

27. *Definition:* *The diagonals in a rectangle are congruent.* Prove that this is true for Quadrilateral 6.

BD ≅ AC

$BD = (-2,3)(3,-2) \quad \sqrt{(3+2)^2+(-2-3)^2} = \sqrt{25+25} = \sqrt{50} \quad 5\sqrt{2}$

$AC = (-3,0)(4,1) \quad \sqrt{(4+3)^2+(1-0)^2} = \sqrt{49+1} = \sqrt{50} \quad 5\sqrt{2}$ ✓